

AD-A063 773

FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO

F/G 1/3

THE UTILIZATION OF THE ANALYTIC GRAPH METHOD TO DETERMINE THE B--ETC(U)

JUN 78 A NASTASE, C NASTASE

UNCLASSIFIED

FTD-ID(RS)T-0977-78

NL

| OF |

AD  
A063773



END  
DATE  
FILMED  
3-79  
DDC

①

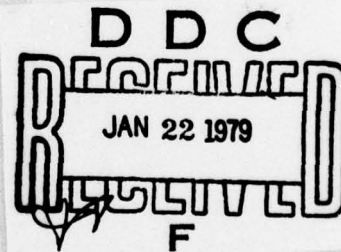
FOREIGN TECHNOLOGY DIVISION



THE UTILIZATION OF THE ANALYTIC GRAPH METHOD  
TO DETERMINE THE BEST SHAPE OF THIN  
DELTA WINGS IN SUPERSONIC FLOW

By

iana Nastase, C. Nastase, and Francine Atanasiu-Moldovan



Approved for public release;  
unlimited.

78 12 26 474

AD-A063773

## EDITED TRANSLATION

FTD-ID(RS)T-0977-78

29 June 1978

MICROFICHE NR: *FTD-78-C-000875*

THE UTILIZATION OF THE ANALYTIC GRAPH METHOD  
TO DETERMINE THE BEST SHAPE OF THIN DELTA  
WINGS IN SUPERSONIC FLOW

By: Adriana Nastase, C. Nastase, and Francine  
Atanasiu-Moldovan

English pages: 16

Source: Studii Si Cercetari De Mecanica  
Aplicata, Vol 32, nr 6, 1973,  
pp. 1179-1190

Country of Origin: Rumania  
Translated by: LINGUISTIC SYSTEMS, INC.  
F33657-76-D-0389  
Patrick T. Nolan

Requester: FTD/TQTA  
Approved for public release;  
distribution unlimited.

ACCESSION BY	
FTD	Whole Section <input checked="" type="checkbox"/>
DDI	Ref Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
REF	AVAIL. and/or SPECIAL
A	

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP-AFB, OHIO.

## STUDIES

### THE UTILIZATION OF THE ANALYTIC GRAPH METHOD TO DETERMINE THE BEST SHAPE OF THIN DELTA WINGS IN SUPERSONIC ~~CURRENTS~~<sup>FLOW</sup>\*

by ADRIANA NASTASE\*\*

C. NASTASE\*\*\*

FRANCINE ATANASIU-MOLDOVAN\*\*\*\*

In the present work an analytic graph method is used (by the first author) <sup>(best)</sup> to determine the best shape of a thin delta wing from among the original thin delta wing class in which the distribution of the vertical speed of disturbance is expressed in the form of second and third order homogenous polynomials. In addition, the thin delta wing satisfies and meets the geometrical and aerodynamic conditions of natural law.

#### 1. INTRODUCTION

The analytic graph method is utilized in the present work (1) for the effective determination, at a given Mach cruising speed  $M$ , the best optimum form of thin delta wings with sub-sonic <sup>leading</sup> edges, which belong to the original class of thin delta

2 wings and which, in addition, satisfy the following conditions of law: the lift power and moment of pitch are given, and the <sup>(axis)</sup> of disturbance speed  $u$  is finite <sup>(leading)</sup> along the subsonic edge of the wings, in order to avoid the formation and falling of the vortexes which have the tendency to appear along that edge at the Mach number of cruising speed  $M$ . The incidents of the thin delta wing are presupposed to be sufficiently small, in such a manner that we are able to apply the results of disturbance theories.

The vertical speeds of disturbance on the wings,  $w$ , is analyzed by using second and third order homogenous polynomial tests.

---

\* The authors thank Mrs. Denise Vallee-Guiraud, main scientific researcher and Mr. M. Bois, scientific researcher at ONERA (France) for valuable advice in laying out the calculus program.

\*\* Bucharest Polytechnical Institute

\*\*\* Galati Polytechnical Institute (honor professor)

\*\*\*\* IMFCA

ST. CERC. MEC. APL., TOM. 32, NR. 6, 1973, P. 1179 - 1190

---

Thus we can apply the results of P. Germain's higher order cone flow theories, (3).

The axis of disturbance speeds  $u$  on the wing is obtained with the help of the hydrodynamic analogy method, through the superimposition of the contributions of the borders and edges on the wing (4), (5).

The study of the optimum form of a thin delta wing through the analytic graph method leads to the determination of the optimum values  $\gamma_0$  of the parameter of similitude  $\gamma = Bl$  of flow, which minimizes the wave resistance expression,  $C_d = f(\gamma)$ . For that purpose an imaginary transformed thin delta wing (projected in a fixed plane) is used, which is the only function of the parameter of similitude  $\gamma$ .

The transformed thin delta wing is placed in a suitably chosen supersonic flow.

## 2. Axial and Vertical Disturbance Velocity Expressions

The thin delta wing is referred to a triorthogonal system of axis  $Ox_1x_2x_3$ , which has the origin in the peak of the wing, axis  $Ox_1$  parallel with the velocity at infinity  $U_\infty$ , again the wing is plotted and numbered in plane  $Ox_1x_2$  (fig.1).

The vertical disturbance velocity  $w_a = \frac{w}{U_\infty}$  on the initial thin delta wing is presupposed as expressed in the form of

second and third order homogenous polynomials

$$w_0 = \frac{w}{U_\infty} = x_1(w_{10} + w_{01}|y|) + x_1^2(w_{20} + w_{11}|y| + w_{02}y^2). \quad (1)$$

In the following similarity transformations (1),

$$\tilde{x}_1 = \frac{x_1}{h_1}, \quad \tilde{x}_2 = \frac{x_2}{l_1}, \quad \tilde{x}_3 = \frac{x_3}{h_1} \quad \left( \tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1} \right), \quad (2)$$

plane  $\tilde{O}\tilde{x}_1\tilde{x}_2$ , the designated transformed plane, corresponds to the initial plain  $Ox_1x_2$ .

The initial thin delta wing projection in plane  $Ox_1x_2$  is an isosceles triangle  $OA_1A_2$  in which the height corresponding to peak  $O$  is  $h_1$ , and its base is  $2\tilde{x}_1^L$  (fig.1).

In the succeeding similarity transformation (2) the transformed thin delta wing projection in plane  $\tilde{O}\tilde{x}_1\tilde{x}_2$  is an isosceles triangle  $\tilde{O}\tilde{A}_1\tilde{A}_2$  which has height 1 and base 2 (fig.2), and the vertical disturbance velocity  $w_a = \tilde{w}_a$  acquires the following form

$$\tilde{w}_0 = \frac{\tilde{w}}{\tilde{U}_\infty} = \tilde{x}_1(\tilde{w}_{10} + \tilde{w}_{01}|\tilde{y}|) + \tilde{x}_1^2(\tilde{w}_{20} + \tilde{w}_{11}|\tilde{y}| + \tilde{w}_{02}\tilde{y}^2). \quad (3)$$

Coefficients  $w_{1j}$  and  $\tilde{w}_{1j}$  from (1) and (3) are connected through the relations:

$$\tilde{w}_{10} = h_1 w_{10}, \quad \tilde{w}_{01} = h_1 l_1 w_{01}, \quad (4)$$

$$\tilde{w}_{20} = h_1^2 w_{20}, \quad \tilde{w}_{11} = h_1^2 l_1 w_{11}, \quad \tilde{w}_{02} = h_1^2 l_1^2 w_{02}. \quad (5)$$

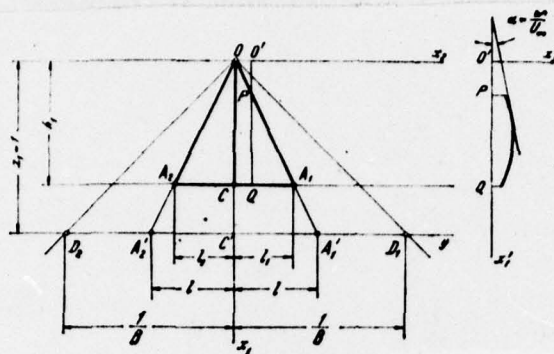


Fig. 1

The expression of the disturbance axis velocity  $u$  on the initial thin delta wing with subsonic leading edges, corresponding to the distribution (1) of the vertical disturbance speed  $w$ , is of the form:

$$u = \frac{u}{U_\infty} = x_1 \left( \frac{A_{20} + A_{22}y^2}{\sqrt{l^2 - y^2}} \right) + x_1^2 \left( \frac{A_{30} + A_{32}y^2}{\sqrt{l^2 - y^2}} + C_{22}y^2 \operatorname{argch} \sqrt{\frac{l^2}{y^2}} \right). \quad (6)$$

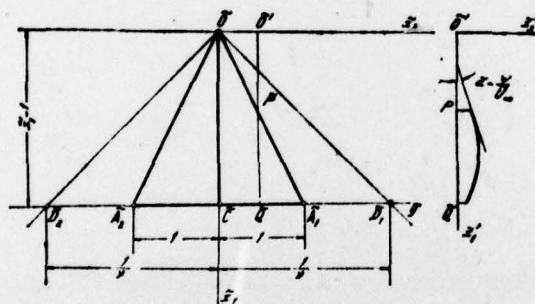


Fig. 2

In a similar manner, the expression of the disturbance axis velocity  $\tilde{u}$  on the transformed thin delta wing is of the form

$$\tilde{u} = \frac{\tilde{u}}{\tilde{U}_\infty} = \tilde{x}_1 \left( \frac{\tilde{A}_{20} + \tilde{A}_{22}\tilde{y}^2}{\sqrt{1 - \tilde{y}^2}} \right) + \tilde{x}_1^2 \left( \frac{\tilde{A}_{30} + \tilde{A}_{32}\tilde{y}^2}{\sqrt{1 - \tilde{y}^2}} + \tilde{C}_{22}\tilde{y}^2 \operatorname{argch} \sqrt{\frac{1}{\tilde{y}^2}} \right). \quad (7)$$

The transformed thin delta wing is presupposed to be placed in an imaginary supersonic flow whose velocity is  $\bar{U}_\infty = \sqrt{1 + v^2}$ .

The axis of disturbance speeds  $u$  and  $\tilde{u}$  are connected through the relation (1)

$$u = \lambda \tilde{u},$$

and among the constants  $A_{1j}$ ,  $C_{1j}$  and  $\tilde{A}_{1j}$ ,  $\tilde{C}_{1j}$  respectively of that axis of disturbance velocity exists relations of the form:

$$\tilde{A}_{20} = \frac{h_1}{j^2} A_{10}; \quad \tilde{A}_{22} = h_1 A_{22}, \quad (9a)$$

$$\tilde{A}_{30} = \frac{h_1^2}{j^2} A_{30}; \quad \tilde{A}_{32} = h_1^2 A_{32}, \quad \tilde{C}_{32} = h_1^2 C_{32}. \quad (9b)$$

The constants  $\tilde{A}_{1j}$  and  $\tilde{C}_{1j}$  of the axis of disturbance velocity  $\tilde{u}$  on the transformed thin delta wing are connected by coefficients  $\tilde{w}_{1j}$  of the vertical disturbance speed  $\tilde{w}$  on that wing through homogenous and linear relations deduced from the compatibility conditions of P. Germain (3):

$$\tilde{A}_{20} = \tilde{a}_{00}^{(2)} \tilde{w}_{10} + a_{01}^{(2)} \tilde{w}_{01}, \quad (10a)$$

$$\tilde{A}_{22} = \tilde{a}_{20}^{(2)} \tilde{w}_{10} + a_{21}^{(2)} \tilde{w}_{01}, \quad (10b)$$

$$\tilde{A}_{30} = \tilde{a}_{00}^{(3)} \tilde{w}_{20} + \tilde{a}_{01}^{(3)} \tilde{w}_{11} + \tilde{a}_{02}^{(3)} \tilde{w}_{02}, \quad (10c)$$

$$\tilde{A}_{31} = \tilde{a}_{10}^{(3)} \tilde{w}_{20} + \tilde{a}_{11}^{(3)} \tilde{w}_{11} + \tilde{a}_{12}^{(3)} \tilde{w}_{02}, \quad (10d)$$

$$\tilde{C}_{32} = \tilde{a}_{21}^{(3)} \tilde{w}_{11}. \quad (10e)$$

If  $\gamma = B\sqrt{v}$  as noted earlier and the elliptic integrals of the first and second cases  $E(k)$  and  $K(k)$ , by the mode

$$k = \sqrt{1 - v^2}, \quad (11)$$

then the coefficients  $a_{00}^{(2)}, a_{01}^{(2)}, c_{01}^{(2)}$  which intervene in the expressions (10 a, b, c, d, e) of the constants of the axis of disturbance velocity  $\tilde{u}$  on the transformed thin delta wing with subsonic leading edges are, respectively, of the form (6), (7), (8):

$$\tilde{a}_{00}^{(2)} = -\frac{2(1-v^2)}{N_2}, \quad (12a)$$

$$\tilde{a}_{01}^{(2)} = -\frac{2[E(k) - v^2 K(k)]}{\pi N_2}, \quad (12b)$$

$$a_{10}^{(2)} = \frac{1-v^2}{N_2}, \quad (12c)$$

$$a_{21}^{(2)} = \frac{2v^2[E(k) - K(k)]}{\pi N_2}, \quad (12d)$$

The following expression is noted with  $N_2$ :

$$N_2 = (1 - 2v^2) E(k) + v^2 K(k), \quad (13)$$

and

$$\tilde{a}_{00}^{(2)} = \frac{2}{N_2} [2(3 - 5v^2 + v^4) E(k) - v^2(3 - 5v^2) K(k)], \quad (14a)$$

$$\tilde{a}_{01}^{(2)} = \frac{6}{\pi N_2} [(4 + v^2) E^2(k) - 8v^2 E(k) K(k) + 3v^4 K^2(k)], \quad (14b)$$

$$\tilde{a}_{11}^{(2)} = -\frac{2}{N_2} [(1 + v^2) E(k) - 2v^2 K(k)], \quad (14c)$$

$$\tilde{a}_{20}^{(2)} = -\frac{2}{N_3} [(4 - 7v^2 + v^4) E(k) - 2(1 - 2v^2) K(k)], \quad (14d)$$

$$\begin{aligned} \tilde{a}_{21}^{(2)} = & -\frac{2}{\pi N_3} [(12 - 17v^2 + 10v^4) E^2(k) - \\ & - 4v^2(2 - v^2 + v^4) E(k)K(k) + v^4(1 + 2v^2) K^2(k)], \end{aligned} \quad (14e)$$

$$\tilde{a}_{22}^{(2)} = \frac{2v^2}{N_3} [2(2 - v^2)E(k) - (3 - v^2)K(k)], \quad (14f)$$

$$c_{21}^{(2)} = -\frac{1}{\pi}, \quad (14g)$$

The expression noted with  $N_3$

$$N_3 = (4 - 19v^2 + 4v^4)E^2(k) + 8v^2(1 + v^2)E(k)K(k) - 5v^4K^2(k). \quad (15)$$

The explicit forms of the relations of connections which occur in problems of the optimum transformed thin delta wing are the following:

- Lift conditions of the transformed thin delta wing to be given as

$$\begin{aligned} \tilde{C}_l = 8 \int \tilde{u}_a \tilde{x}_1 d\tilde{x}_1 d\tilde{y} = & \tilde{\Lambda}_{20} \tilde{w}_{10} + \tilde{\Lambda}_{21} \tilde{w}_{01} + \tilde{\Lambda}_{20} \tilde{w}_{20} + \\ & + \tilde{\Lambda}_{31} \tilde{w}_{11} + \tilde{\Lambda}_{32} \tilde{w}_{02} = \frac{2\pi}{3} [(2\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)}) \tilde{w}_{10} + \\ & + (2\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)}) \tilde{w}_{01}] + \frac{\pi}{2} [(2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}) \tilde{w}_{20} + (2\tilde{a}_{01}^{(3)} + \\ & + \tilde{a}_{21}^{(3)} + \frac{c_{21}^{(3)}}{3}) \tilde{w}_{11} + (2\tilde{a}_{02}^{(3)} + \tilde{a}_{21}^{(3)}) \tilde{w}_{02}] = \tilde{C}_l = \frac{C_{l_0}}{l}. \end{aligned} \quad (16)$$

The moment of pitch condition of the transformed thin delta wing to be given as:

$$\begin{aligned}
\tilde{O}_m = 8 \int \tilde{u}_a \tilde{x}_1^2 d\tilde{x}_1 d\tilde{y} = & \tilde{\Gamma}_{20} \tilde{w}_{10} + \tilde{\Gamma}_{21} \tilde{w}_{01} + \tilde{\Gamma}_{30} \tilde{w}_{20} + \tilde{\Gamma}_{31} \tilde{w}_{11} + \\
& + \tilde{\Gamma}_{32} \tilde{w}_{02} = \frac{3}{4} (\tilde{\Lambda}_{20} \tilde{w}_{10} + \tilde{\Lambda}_{21} \tilde{w}_{01}) + \frac{4}{5} (\tilde{\Lambda}_{30} \tilde{w}_{20} + \tilde{\Lambda}_{31} \tilde{w}_{11} + \\
& + \tilde{\Lambda}_{32} \tilde{w}_{02}) = \tilde{O}_{m_0} = \frac{C_{m_0}}{i}.
\end{aligned} \quad (17)$$

The disturbance axis velocity condition  $\tilde{u}$  is to be finite along the subsonic leading edges of the transformed wings (1) leading in this case to the relation of connection of the form:

$$\begin{aligned}
F_2 \equiv \tilde{A}_{20} + \tilde{A}_{22} &= 0, & (18a) \\
F_3 \equiv \tilde{A}_{30} + \tilde{A}_{32} &= 0. & (18b)
\end{aligned}$$

If relations (10a,b,c,d,e) are taken into consideration, these relations are written in the form:

$$F_2 \equiv \tilde{T}_{20} \tilde{w}_{10} + \tilde{T}_{21} \tilde{w}_{01} = (\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)}) \tilde{w}_{10} + (\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)}) \tilde{w}_{01} = 0, \quad (19)$$

$$\begin{aligned}
F_3 \equiv \tilde{T}_{30} \tilde{w}_{20} + \tilde{T}_{31} \tilde{w}_{11} + \tilde{T}_{32} \tilde{w}_{02} = & (\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}) \tilde{w}_{20} + \\
& + (\tilde{a}_{01}^{(3)} + \tilde{a}_{21}^{(3)}) \tilde{w}_{11} + (\tilde{a}_{02}^{(3)} + \tilde{a}_{22}^{(3)}) \tilde{w}_{02} = 0.
\end{aligned} \quad (20)$$

The expression of the coefficient of wave resistance of the transformed thin delta wing is in the form:

$$\begin{aligned}
C_d = 8 \int_{\partial \Delta} \tilde{w}_a \tilde{u}_a \tilde{x}_1 d\tilde{x}_1 d\tilde{y} = & \tilde{\Omega}_{2200} \tilde{w}_{10}^2 + (\tilde{\Omega}_{2210} + \\
& + \tilde{\Omega}_{2201}) \tilde{w}_{10} \tilde{w}_{01} + \tilde{\Omega}_{2211} \tilde{w}_{01}^2 + (\tilde{\Omega}_{2300} + \tilde{\Omega}_{2300}) \tilde{w}_{10} \tilde{w}_{01} + \\
& + (\tilde{\Omega}_{2301} + \tilde{\Omega}_{2310}) \tilde{w}_{01} \tilde{w}_{20} + (\tilde{\Omega}_{2311} + \tilde{\Omega}_{2311}) \tilde{w}_{01} \tilde{w}_{11} + \\
& + (\tilde{\Omega}_{2321} + \tilde{\Omega}_{2312}) \tilde{w}_{01} \tilde{w}_{02} + \tilde{\Omega}_{2300} \tilde{w}_{20}^2 + \tilde{\Omega}_{2311} \tilde{w}_{11}^2 +
\end{aligned}$$

$$+ \tilde{\Omega}_{3322} \tilde{w}_{02}^2 + (\tilde{\Omega}_{3310} + \tilde{\Omega}_{3301}) \tilde{w}_{20} \tilde{w}_{11} + (\tilde{\Omega}_{3320} + \\ + \tilde{\Omega}_{3302}) \tilde{w}_{20} \tilde{w}_{02} + (\tilde{\Omega}_{3321} + \tilde{\Omega}_{3312}) \tilde{w}_{11} \tilde{w}_{02},$$

$$\left( \tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1} \right).$$

(21)

Continuing, the following definite integral is marked with  $\tilde{\mathcal{J}}_k$ :

$$\tilde{\mathcal{J}}_k = \int_0^1 \frac{\tilde{y}^k d\tilde{y}}{\sqrt{1-\tilde{y}^2}}, \quad (22a)$$

$$\tilde{\mathcal{J}}_{2t} = \frac{\pi(2t)!}{2^{2t+1}(t!)^2} \quad (k = 2t), \quad (22b)$$

$$\tilde{\mathcal{J}}_{2t+1} = \frac{2^{2t}(t!)^2}{(2t+1)!} \quad (k = 2t+1). \quad (22c)$$

From which for  $k = 0, 1, 2, 3, 4, 5$  the following values for  $\mathcal{J}_k$  result:

$$\mathcal{J}_0 = \frac{\pi}{2}, \quad \mathcal{J}_1 = 1, \quad (23a)$$

$$\mathcal{J}_2 = \frac{\pi}{4}, \quad \mathcal{J}_3 = \frac{2}{3}, \quad (23b)$$

$$\mathcal{J}_4 = \frac{3\pi}{16}, \quad \mathcal{J}_5 = \frac{8}{15}. \quad (23c)$$

With these notations  $\tilde{\Omega}_{nmkj}$ , the constants, which appear in the expression (21) of wave resistance  $\tilde{\mathcal{C}}_d$  of the transformed thin delta wing can be written under the form

$$\tilde{\Omega}_{2mkj} = \frac{8}{m+2} (\tilde{a}_{0j}^{(2)} \tilde{\mathcal{J}}_k + \tilde{a}_{2j}^{(2)} \tilde{\mathcal{J}}_{k+2}) \quad (24)$$

$$(m = 2, 3, \quad k = 0, 1, \dots, (m-1), \quad j = 0, 1)$$

and, respectively,

$$\tilde{\Omega}_{3mkj} = \frac{8}{m+3} \left[ \tilde{a}_{0j}^{(3)} \tilde{\sigma}_k + \left( \tilde{a}_{2j}^{(3)} + \frac{\tilde{\sigma}_{2j}^{(3)}}{k+3} \right) \tilde{\sigma}_{k+2} \right] \quad (25)$$

$$(m = 2, 3 \quad k = 0, 1, \dots, (m-1), \quad j = 0, 1, 2).$$

To explain by means of example, in this case, the coefficients are in the form:

$$\Omega_{2200} = \frac{\pi}{2} (2\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)}), \quad \tilde{\Omega}_{2210} = \frac{2}{3} (3\tilde{a}_{00}^{(2)} + 2\tilde{a}_{20}^{(2)}), \quad (26a)$$

$$\tilde{\Omega}_{2201} = \frac{\pi}{2} (2\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)}), \quad \tilde{\Omega}_{2211} = \frac{2}{3} (3\tilde{a}_{01}^{(2)} + 2\tilde{a}_{21}^{(2)}) \quad (26b)$$

for  $n = 2$  and  $m = 2$ ,

$$\tilde{\Omega}_{2300} = \frac{2\pi}{5} (2\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)}), \quad \tilde{\Omega}_{2310} = \frac{8}{15} (3\tilde{a}_{00}^{(2)} + 2\tilde{a}_{20}^{(2)}), \quad (27a)$$

$$\tilde{\Omega}_{2301} = \frac{2\pi}{5} (2\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)}), \quad \tilde{\Omega}_{2311} = \frac{8}{15} (3\tilde{a}_{01}^{(2)} + 2\tilde{a}_{21}^{(2)}), \quad (27b)$$

$$\tilde{\Omega}_{2320} = \frac{\pi}{16} (4\tilde{a}_{00}^{(2)} + 3\tilde{a}_{20}^{(2)}), \quad (27c)$$

$$\tilde{\Omega}_{2321} = \frac{3\pi}{10} (4\tilde{a}_{01}^{(2)} + 3\tilde{a}_{21}^{(2)}) \quad (27d)$$

for  $n = 2$  and  $m = 3$ ,

$$\tilde{\Omega}_{3200} = \frac{2\pi}{5} (2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}), \quad \tilde{\Omega}_{3210} = \frac{8}{15} (3\tilde{a}_{00}^{(3)} + 2\tilde{a}_{20}^{(3)}), \quad (28a)$$

$$\tilde{\Omega}_{3201} = \frac{2\pi}{15} (6\tilde{a}_{01}^{(3)} + 3\tilde{a}_{11}^{(3)} + \tilde{\sigma}_{11}^{(3)}), \quad \tilde{\Omega}_{3211} = \frac{4}{15} (6\tilde{a}_{01}^{(3)} + 4\tilde{a}_{11}^{(3)} + \tilde{\sigma}_{11}^{(3)}), \quad (28b)$$

$$\tilde{\Omega}_{3202} = \frac{2\pi}{5} (2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}), \quad \tilde{\Omega}_{3212} = \frac{8}{15} (3\tilde{a}_{02}^{(3)} + 2\tilde{a}_{22}^{(3)}), \quad (28c)$$

for  $n = 3$  and  $m = 2$  respectively,

$$\tilde{\Omega}_{3300} = \frac{2\pi}{5} (2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}), \quad \tilde{\Omega}_{3310} = \frac{4}{9} (3\tilde{a}_{00}^{(3)} + 2\tilde{a}_{20}^{(3)}), \quad (29a)$$

$$\tilde{\Omega}_{3302} = \frac{\pi}{3} (2\tilde{a}_{02}^{(3)} + \tilde{a}_{22}^{(3)}), \quad \tilde{\Omega}_{3311} = \frac{2}{9} (6\tilde{a}_{01}^{(3)} + 4\tilde{a}_{11}^{(3)} + \tilde{\sigma}_{11}^{(3)}), \quad (29b)$$

$$\tilde{\Omega}_{3320} = \frac{\pi}{12} (4\tilde{a}_{00}^{(3)} + 3\tilde{a}_{20}^{(3)}), \quad (29d)$$

$$\tilde{\Omega}_{3321} = \frac{\pi}{60} (20\tilde{a}_{01}^{(3)} + 15\tilde{a}_{21}^{(3)} + 3\tilde{a}_{41}^{(3)}), \quad (29e)$$

$$\tilde{\Omega}_{3322} = \frac{\pi}{12} (4\tilde{a}_{02}^{(3)} + 3\tilde{a}_{22}^{(3)}) \quad (29f)$$

for  $n = 3$  and  $m = 3$ .

The equations (1), (9) obtained through canceling all of the variation of coefficients  $\delta \tilde{w}_{0q}$  are, in this case, of the form:

$$\begin{aligned} 2\tilde{\Omega}_{2200}\tilde{w}_{10} + (\tilde{\Omega}_{2201} + \tilde{\Omega}_{2210})\tilde{w}_{01} + (\tilde{\Omega}_{2200} + \tilde{\Omega}_{2200})\tilde{w}_{20} + \\ + (\tilde{\Omega}_{2201} + \tilde{\Omega}_{2210})\tilde{w}_{11} + (\tilde{\Omega}_{2202} + \tilde{\Omega}_{2220})\tilde{w}_{02} + \lambda^{(1)}\tilde{A}_{20} + \\ + \lambda^{(2)}\tilde{T}_{20} + \tilde{\lambda}_2\tilde{T}_{20} = 0, \end{aligned} \quad (30)$$

$$\begin{aligned} (\tilde{\Omega}_{2210} + \tilde{\Omega}_{2201})\tilde{w}_{10} + 2\tilde{\Omega}_{2211}\tilde{w}_{01} + (\tilde{\Omega}_{2212} + \tilde{\Omega}_{2212})\tilde{w}_{20} + (\tilde{\Omega}_{2211} + \\ + \tilde{\Omega}_{2211})\tilde{w}_{11} + (\tilde{\Omega}_{2212} + \tilde{\Omega}_{2221})\tilde{w}_{02} + \lambda^{(1)}\tilde{A}_{21} + \lambda^{(2)}\tilde{T}_{21} + \\ + \tilde{\lambda}_2\tilde{T}_{21} = 0, \end{aligned} \quad (31)$$

$$\begin{aligned} (\tilde{\Omega}_{2300} + \tilde{\Omega}_{2300})\tilde{w}_{10} + (\tilde{\Omega}_{2301} + \tilde{\Omega}_{2310})\tilde{w}_{01} + 2\tilde{\Omega}_{2300}\tilde{w}_{20} + \\ + (\tilde{\Omega}_{2301} + \tilde{\Omega}_{2310})\tilde{w}_{11} + (\tilde{\Omega}_{2302} + \tilde{\Omega}_{2320})\tilde{w}_{02} + \lambda^{(1)}\tilde{A}_{20} + \\ + \lambda^{(2)}\tilde{T}_{20} + \tilde{\lambda}_2\tilde{T}_{20} = 0, \end{aligned} \quad (32)$$

$$\begin{aligned} (\tilde{\Omega}_{2310} + \tilde{\Omega}_{2301})\tilde{w}_{10} + (\tilde{\Omega}_{2311} + \tilde{\Omega}_{2311})\tilde{w}_{01} + (\tilde{\Omega}_{2310} + \tilde{\Omega}_{2301})\tilde{w}_{20} + \\ + 2\tilde{\Omega}_{2311}\tilde{w}_{11} + (\tilde{\Omega}_{2312} + \tilde{\Omega}_{2321})\tilde{w}_{02} + \lambda^{(1)}\tilde{A}_{21} + \lambda^{(2)}\tilde{T}_{21} + \\ + \tilde{\lambda}_2\tilde{T}_{21} = 0, \end{aligned} \quad (33)$$

$$\begin{aligned} (\tilde{\Omega}_{2320} + \tilde{\Omega}_{2302})\tilde{w}_{10} + (\tilde{\Omega}_{2321} + \tilde{\Omega}_{2312})\tilde{w}_{01} + (\tilde{\Omega}_{2320} + \tilde{\Omega}_{2302})\tilde{w}_{20} + \\ + (\tilde{\Omega}_{2321} + \tilde{\Omega}_{2312})\tilde{w}_{11} + 2\tilde{\Omega}_{2322}\tilde{w}_{02} + \lambda^{(1)}\tilde{A}_{22} + \lambda^{(2)}\tilde{T}_{22} + \\ + \tilde{\lambda}_2\tilde{T}_{22} = 0. \end{aligned} \quad (34)$$

The equations (30 - 34), together with the relations of connection (16), (17), (18a,b), (19) and (20), form a linear system

and

## APPENDIX 1

and  $\checkmark \pi_i \tilde{\lambda}_3$

to be a rectangular matrix  $R$  of the type  $m \times n = 9 \times 2$ ,

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ \vdots & \vdots \\ R_{91} & R_{92} \end{pmatrix}, \quad (35) \quad (35)$$

in which  $R_{ij} = 0$  with the exception of the terms  $R_{61} = 1$  and  $R_{72} = 1$ . If the solutions of the system obtained for the two sets of values for linear terms  $R_{11}$  and  $R_{12}$  is noted with  $\tilde{w}'_{ij}$  and  $\tilde{w}''_{ij}$  ( $i = 1, \dots, 9$ ) then the optimum values of the coefficients  $\tilde{w}_{ij}$  have the values given by the relation

$$\tilde{w}_{ij} = \frac{C_{L1} B}{v} \tilde{w}'_{ij} + \frac{C_{L2} B}{v} \tilde{w}''_{ij}. \quad (36) \quad (36)$$

For the determination of the optimum form of projection in a plane at the Mach flight number of  $M = 2$ , the range of admissible values of the span of the wing  $\lambda = \frac{l_{21}}{h_1}$  the range of values contained between  $0,2 \leq \lambda \leq 0,55$  is considered (the value  $\lambda = 0,5774$  corresponds to the sonic leading edge for Mach cruising speed  $M = 2$ ).

Therefore, the following range of values admissible for  $\lambda$  result :

$$0,3464 \leq \lambda \leq 0,9526. \quad (37)$$

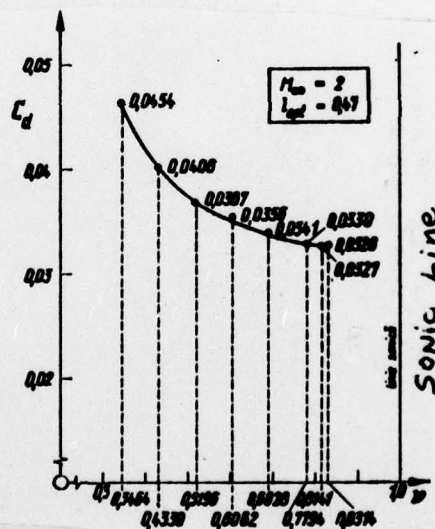
A set of values of the parameter of similitude  $\lambda$  contained in this range of admissible values is considered, with the help of

the following,  $(C_d)_{opt}$  and the curve  $(C_d)_{eight} = f(v)$  (for  $B = \text{constant}$ ) which has the aspect in figure 3, is drawn.

With the help of this curve the optimal value of the parameter of similitude  $\chi$  is graphically determined (for which this curve touches its minimum).

The optimum value  $\chi_0$  of the parameter of similitude  $\chi$ , together with the given area of projection in the plane, determines the optimum form of the initial thin delta wings' projection on the plane.

Figure 3



To make it clear, consider if you will, the following given initially:

- the number of Mach speed,  $M_{\infty} = 2$ ,
- the coefficient of lift,  $C_{\chi} = 0.3$ ,
- the coefficient of the moment of pitch,  $C_m = 0.23$ ,
- the area of projection of the plane,  $A_0 = 75 \text{ m}^2$ ,

the optimum form of the projection on the plane of the initial thin delta wing is the isosceles triangle which has a height of  $h_1 = 12.63$  m and a base of  $2\lambda_1^l = 11.86$  m, and the optimum unit span is  $\lambda = \frac{\lambda_1^l}{h_1} \approx 0.47$ .

The minimum value of wave resistance of the initial thin delta wing is given by the relation:

$$C_d = \tilde{C}_d = 0.327.$$

(3c)

Accepted by the editors on Dec. 29, 1972

#### BIBLIOGRAPHY

- 1) NĂSTASE ADRIANA, *Eine graphisch-analytische Methode zur Bestimmung der optimum optimorum-Form des dünnen Deltaflügels in Übetschallströmung*, RRST Mec Appl. 18, 4 (1973).
- 2) LEGENDRE R., *Nappes de tourbillons en cornets aux bords d'attaque d'une aile en delta*, La Rech. Aeron. 70, Mai 1959.
- 3) GERMAIN P., *La théorie des mouvements homogènes et son application au calcul de certaines ailes delta en régime supersonique*, Rech. Aeron. 7, 3-16 (1949).
- 4) CARAFOLI E., IONESCU M., *Écoulements coniques d'ordre supérieur autour des ailes triangulaires où à épaisseur symétrique*, Rev. Méc. Appl. 2, 1 (1957).
- 5) CARAFOLI E., MATEESCU D., NĂSTASE ADRIANA, *Wing Theory in Supersonic Flow*, cap. 11 Pergamon Press (1969).
- 6) NĂSTASE ADRIANA, *Forme aerodinamice optime prin metoda variațională (cap. IV)*. Edit. Academiei, 1969.
- 7) CARAFOLI E., NĂSTASE ADRIANA, *Étude des ailes triangulaires minces à symétrie forcée*, Rev. Méc. Appl. 3, 4 (1958).
- 8) CARAFOLI E., NĂSTASE ADRIANA, *Minimum Drag Thin Delta Wing with Finite Velocity at the Leading Edges, in Supersonic Flow*, Rev. Méc. Appl. 11, 2 (1967).
- 9) CARAFOLI E., NĂSTASE ADRIANA, *Minimum Drag Thin Triangular Wing in Supersonic Flow*, Rev. Méc. Appl. 5, 4 (1960).

- 1) NASTASE ADRIANA, A graphical-analysis method of determining the best shape of the thin delta wing in supersonic flow, RRST Mech. Appl. 18, 4 (1973).
- 2) LEGENDRE R., Rolled-up vortex sheets at the leading edges of a delta wing, La Rech. Aeron. 70, May 1959.
- 3) GERMAINE P., The theory of homogeneous movements and its application to the calculation of certain delta wings at supersonic speed. Rech. Aeron. 7, 3-16 (1949).
- 4) CARAFOLI E., IONESCU M., High-order conical flows about the triangular or symmetrically thick wing. Rev. Mec. Appl. 2, (1957).
- 5) CARAFOLI E., MATEESCU D, NASTASE ADRIANA, Wing Theory in Supersonic Flow, cap. 11., Pergamon Press (1969).
- 6) NASTASE ADRIANA, Optimal Aerodynamic Form Using the Variational Method.
- 7) CARAFOLI E., NASTASE ADRIANA, Study of Thin Triangular Forced Symmetry Wings, Rev. Mec. Appl 3, 4 (1958).
- 8) CARAFOLI E., NASTASE ADRIANA, Minimum Drag Thin Delta Wing with Finite Velocity at the Leading Edges, in Supersonic Flow, Rev. Mech Appl. 11, 2 (1967).
- 9) CARAFOLI E., NASTASE ADRIANA, Minimum Drag Thin Triangular Wing in Supersonic Flow, Rev. Mec. Appl. 5, 5 (1960).

# DISTRIBUTION LIST

## DISTRIBUTION DIRECT TO RECIPIENT

<u>ORGANIZATION</u>	<u>MICROFICHE</u>	<u>ORGANIZATION</u>	<u>MICROFICHE</u>
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
B344 DIA/RDS-3C	8	E403 AFSC/INA	1
C043 USAMIIA	1	E404 AEDC	1
C509 BALLISTIC RES LABS	1	E408 AFWL	1
C510 AIR MOBILITY R&D	1	E410 ADTC	1
LAB/FIO	.	E413 ESD	2
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NICD	3
C619 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NICD	2
H300 USAICE (USAREUR)	1		
P005 ERDA	1		
P005 CIA/CRS/ADB/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		